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TRANSMISSION OF FERTILITY PATTERN IN MOTHER-DAUGHTER RELATION – BAYESIAN VIEW (A CASE STUDY OF AUSTRIA)[1]

INTRODUCTION

Intergenerational transmission of reproductive patterns has interested demographers and representatives of other sciences since the end of the nineteenth century. Pioneers in this field were Pearson, Lee and Bramley-Moore, who analyzed the correlation between fertility in subsequent generations among wealthy British population (Pearson and Lee 1899, Pearson et al. 1899). Several relations were investigated: mother-daughter, father-son and grandparents-grandsons/granddaughters. The strongest correlation was observed between fertility of mothers and daughters. Since these studies, scientific interest in reproductive behavior within the family began to grow rapidly, and further studies, in which different communities living in different periods were considered, provide new knowledge and at the same time have revealed new research problems and areas of dependence (see Fisher 1930, Langford and Wilson 1985: 437–443, Anderton et al. 1987: 467–480, Murphy 1987: 463–485, Bocquet-Appel and Jakobi 1993: 335–347). Review of these researches was done by Murphy (Murphy 1999: 122–145). The summarization of those studies revealed that the strongest dependence had occurred among mothers' and daughters' populations. Additionally, the connection between fertility of parents and their children seems to be insignificant among pre-transitional populations, but becomes more important over time, especially in developed countries. What is more, Murphy emphasized

[1] The Polish data was not chosen because of the two following reasons: firstly, the paper was prepared based on the research problem concerning Austria that was discussed during the Vienna Institute of Demography Colloquium in 31 Jan 2013; secondly, the study was conducted when the GGS-PL data were not yet publicly available.

that before the second demographic transition the connection between fertility of parents and children was negligible, because there were no significant correlation coefficients. However, the connection has changed over time and the impact of the fertility pattern in the origin-family home has become more and more important among subsequent generations. Murphy suggested that in developed countries this connection might even have similar importance to other socio-economic determinants, e.g., the education of a woman.

There are two major theoretical approaches that refer to intergenerational transmission of fertility patterns: the Easterlin hypothesis (Easterlin 1978, 1987) and the Low Fertility Trap Hypothesis (LFTH), first proposed by Wolfgang Lutz and Vegard Skirbekk (Lutz and Skirbekk 2005) and developed later together with Maria Rita Testa (Lutz et al. 2006).

Easterlin claimed that fertility is a cyclic, fluctuating process, in which huge generations are more likely to provide a small number of offspring, and vice versa – small ones provide big subsequent generations. The changes in individual family size are observed due to preferences of potential parents which were already shaped during their adolescence. Young adult people compare the quality of their life (often measured by income) with the life of their parents, and, when they think they are more successful, they will be more likely to have a big family. On the other hand, when they find themselves in a worse position than their parents were, they will decide to have fewer children. The fluctuations in the fertility process occur mainly due to different opportunities of individuals living in different environments. The environmental conditions (e.g., labour market) are created by the size of a generation: big generations provide a more competitive, insecure environment in which individuals reduce their fertility, while small generations ensure more peaceful, calmer conditions that consequently encourage individuals to have bigger families.

The Low Fertility Trap Hypothesis aims to explain possible "self-reinforcing" changes in fertility in countries with already low fertility level. The hypothesis consists of three components. The first is purely demographic and is based on a negative momentum: fewer women in the future will provide fewer children. The second one is sociological and claims that when the actual fertility is low, the next generations will "inherit" this low fertility level and the ideal family size among young generations will decline. The third one is economic and is based on the Easterlin hypothesis, mentioned above. It assumes that aspirations of the next generations are increasing, while simultaneously their expected income is declining, and consequently both will result with lower fertility. Authors of the LFTH, through the second and third components, drew major attention to the importance of the transmission of fertility pattern among generations. They warned that no immediate actions taken by the government in low fertility countries will lead to the "downward spiral" in the number of births and an inescapable fertility "trap" (Lutz et al. 2006).

The connection between reproductive behaviors among family still appears as a current problem. Hognas and Carlson in their study from 2012 showed that among the US population, children born to unmarried parents are more likely to have a nonmarital first birth, so the transmission of nonmarital childbearing across generations is observed and it is spreading over the population (Högnäs and Carlson 2012: 1480–94). Tanaka and Iwasa also investigated the transmission of fertility preferences in the case of the evolution of *hinoeuma*[2] superstition in Japan (Tanaka and Iwasa 2012: 20–28). The results clearly showed that there is a strong connection between fertility of children and their mother. In those families where children "inherited" the belief in *hinoeuma* year from their mother, the superstition is continued in the next generation. On the other hand, in families where the father believed in *hinoeuma*, the superstition became weak in the next generation, and finally fully disappears. The multigenerational transmission of fertility patterns among the Swedish population was investigated by Kolk (Kolk 2011, 2013). There was revealed not only the strong positive connection between fertility of parents and children but also some influence of kin's fertility pattern was observed. Additionally, the positive relation between a woman's origin-family size and her own number of children was also revealed among the British contemporary population (Booth and Kee 2009). Similar results, but among the Polish historical population was revealed by Tymicki (Tymicki 2006).

Investigating intergenerational transmission of fertility patterns was highly limited mostly because of difficulties in finding suitable, reliable multigenerational datasets. Nowadays, due to availability of large sample surveys or administrative digitized registers, intergenerational transmission of fertility can be analyzed deeper. So far, to explain the fertility pattern of the previous generation, the number of a respondent's siblings was included in the analyses. However, the study of Martin-Matthews, who analyzed a sample of multigenerational families in Canada in 1995, has shown that there is a strong correlation between a woman's age at childbearing and that of her mother, which consequently influences the daughter's family size (Martin-Matthews et al. 2001). That is why it seems to be reasonable to explain the fertility pattern of the generation of mothers using not only the number of a daughter's siblings but also the age of her mother at childbearing. What is more, in the literature so far there is no distinction between the impact of the fertility pattern of the previous generation separately on childlessness and parenthood. Considering the increasing percentage of childless couples in developed countries, it is worth taking into account both states.

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[2] *Hinoeuma* is one of the sixty possible combinations of zodiac signs – it is the coexistence of the fire and the horse. It takes place every 60 years. Superstition connected with *hinoeuma* says that a woman born in this year will make wrong marital choices and therefore will be a bad wife for her future husband. The superstition in Japan is so strong that it led to significant decrease of number of births in subsequent *hinoeuma* years: 1846, 1906 and 1966.

Following Murphy's suggestion that the transmission of fertility patterns could be very important among highly developed countries, and based on the previous studies that have shown significant connection between female generations, this paper concerns the analyses of Austrian women. Austria is an example of a highly-developed country with still popular traditional gender family roles and much more family-oriented policy than, for example, Nordic countries or the United Kingdom. The fertility level has been stable over the past three decades with the total fertility rates hovering in the range of 1.3 to 1.5. However, Austria is observed to have one of the highest levels of childlessness, nowadays exceeding 20% of woman (Statistics Austria[3], Spielauer 2005, Buber et al. 2012).

The aim of this paper has clearly arisen from the reflections presented above – the main goal is to investigate the intergenerational transmission of fertility in contemporary populations taking as an example the case of the mother-daughter relation in Austria. In other words, the main interest of this study is to determine the effect of demographic variables, which describe the fertility pattern of the mother (number of children and age at childbearing) on the daughter's family model. Simultaneously, several control variables specified for the daughter (age, place of residence, educational level, marital status) were included into the analysis. In the next step, to simulate the reproductive behavior according to levels of adopted characteristics, several female profiles will be constructed.

In surveys on connection between fertility of parents and children it was widely adopted, following Pearson's example, to use simple correlation analyses, mostly because of its simplicity and ease of interpretation. However, this method in more complicated problems does not always provide the proper results. Therefore, in this paper we propose the *Zero-Inflated Poisson* regression model estimated using a Bayesian approach that enables us to make formal inferences about uncertainty (the method was firstly proposed in Osiewalska 2012). Additionally, the ZIP model allows us to analyze childlessness and parenthood as separate states but still connected by the probability of childlessness.

In view of the previous studies and results presented in the literature, within the main hypothesis we assume that family procreative behaviors among contemporary Austrian females are affected by the fertility pattern of the previous generation. The fertility pattern developed in the family home is represented by the number of a woman's siblings and her mother's age at childbearing. Additionally, three detailed hypotheses were formulated. The first one assumes that there is a positive correlation between a woman's origin-family size and the total number of a woman's children. In the second one we assume that procreative behavior of a woman is determined by her mother's age at childbearing – the younger a mother is at the daughter's birth, the bigger the daughter's family. Finally, since becoming a parent for the first time is influenced by the family home conditions, for example, the support from

[3] http://www.statistik.at/web_en/

relatives (Dommermuth et al. 2011), we expect that the fertility pattern of a mother has a stronger impact on the daughter's probability of being childless than on the daughter's expected family size (parenthood).

Data used in this study came from the first wave of the *Generations and Gender Study*[4] for Austria (2008–2009). From the original dataset that consists of 3001 women, 105 respondents were removed due to incomplete information about the respondent or her mother (only 3.5% of initial sample). Finally, 2896 "daughters" were taken into the analysis. This dataset is big enough to perform the formal inference and the lack of 3.5% of initial data does not disturb the results. In the case of a small dataset, when omitting incomplete data would lead to deficient or even false results, Bayesian techniques of interpolation missing data could be useful (Osiewalski and Osiewalski 2012: 169–197). All calculations were done with R project.

This study contributes to the current knowledge both empirically and methodologically. Contemporary female reproductive behaviors in Austria will be investigated in order to find the connection with the fertility patterns of the mothers' generation. Mothers' fertility patterns will be explained not only by the number of children (usually done in previous studies) but also by the mothers' age at a daughter's birth. Additionally, childlessness and parenthood will be distinguished during analysis. This approach will allow us to find new dependencies between mothers' and daughters' fertility patterns, which seems to be particularly important in view of one of the highest level of childlessness in Austria. This is an empirical contribution. On the methodological side, the Zero-Inflated Poisson model will be introduced for the first time in a wider demographic society[5]. It will be applied to the investigation of contemporary reproductive behavior. Therefore, complex analysis with the distinction between childlessness and parenthood implemented in one fertility model will become possible. Additionally, Bayesian approach to the ZIP model specification will be adapted, so it will be possible to incorporate prior knowledge and make precise analysis of uncertainty of the model's parameters.

This paper consists of four sections. Section I is an introduction, whereas in section II models applied in the research are described. Section III presents the estimations of the models and comments on results. Section IV demonstrates general conclusions.

[4] http://www.ggp-i.org/

[5] The first note about the ZIP model in demography was made by Osiewalska in the typescript available in polish at Cracow University of Economics (Osiewalska 2012).

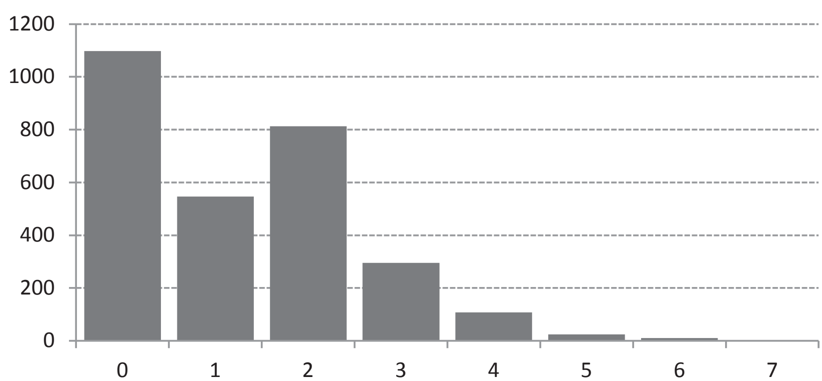
MODELING TRANSMISSION OF FERTILITY PATTERN

ZERO-INFLATED POISSON MODEL

Due to the discrete nature of the analyzed event – the birth of a child (the number of children is a discrete variable) – in fertility analysis we are restricted to the count modeling framework. These models with appropriate specifications allow us to determine the impact of socio-economic factors on the studied phenomenon (Alho and Spencer 2005). The most common of count models is the Poisson distribution (perfect for rare events), which can be easily generalized to the Poisson regression model. In literature there are also other generalizations proposed, which may be useful for more (or less) zeros in datasets than the expected number of zeros in the standard Poisson distribution – they are called Zero-Inflated Poisson (ZIP) models (Lambert 1992: 1–14, Jansakul and Hinde 2001: 75–96).

To choose an appropriate model for the relationship between reproductive behavior of mothers and daughters, we need to take into consideration that among the analyzed population, a large number of childless daughters (zeros) occurs (see Figure 1). It is clear that the number of zeros in the sample exceeds the number which can be explained by standard the Poisson distribution. Besides, the zero value in fertility analysis has a different meaning – childlessness – and it is inappropriate or even wrong to compare a family with zero children to a family with one child in the same way that we compare one child versus two children families. Childlessness is a strongly different state than having at least one child. Therefore, in this study the Zero-Inflated Poisson model was applied. It is important that the specification of the model allows treating childlessness as a qualitatively different state than having children. It means that the ZIP model gives the opportunity to set up other determinants in modeling zero than in modeling counts.

Figure 1. Number of children ever born



The Zero-Inflated Poisson regression model was used with the following form (compare: Lambert 1992: 1–14, Jansakul and Hinde 2001: 75–96, Marzec and Osiewalski 2012). The independent variables $Y = [Y_1,...,Y_n]$ are derived from the Zero-Inflated Poisson distribution. The ZIP distribution for the variable Y_i, denoted by $ZIP(\lambda_i, p_i)$, can be represented as follows:

$$P(Y_i = y_i) = \begin{cases} p_i, \quad y_i = 0 \\ \dfrac{(1-p_i)}{\left(1-\exp(-\lambda_i)\right)}\exp(-\lambda_i)\dfrac{\lambda_i^{y_i}}{y_i!}, \quad y_i = 1,2,..., \quad \lambda_i > 0, \quad p_i \in [0,1]. \end{cases} \tag{1}$$

The ZIP model, as in equation (1), has two states: *zero* with probability p and *count* with probability $(1 - p)$. If the aim of the analysis additionally includes determining the impact of selected variables on the studied phenomenon, the ZIP model for n observations can be modified as follows:

$$p_j = \frac{\exp(x_j\gamma)}{1+\exp(x_j\gamma)}, \ \lambda_j = \exp(w_j\delta),$$
$$j=1,...,n, \tag{2}$$
$$x_j = [x_{j1},...,x_{js}], \ w_j = [w_{j1},...,w_{jr}], \ \gamma = [\gamma_1,...,\gamma_s]', \ \delta = [\delta_1,...,\delta_r]'$$

where x_j, x_j are covariate vectors for observation $j=1,...,n,$ and γ, δ are vectors of parameters for, respectively, zero and count states. This form of parameters p_j and λ_j ensures that the constraints on these parameters hold. It should be noted that p and λ are increasing functions of, respectively, γ and δ for fixed covariate vectors x_j or w_j.

Standard methods for the ZIP model's estimation are relatively simple to use, because of their usually uncomplicated numerical structure, as well as the wide availability in many statistical packages (e.g., in R project in *pscl* library). However, these methods do not always provide the expected results. The first problem arises when it is numerically complex to find the global extremum of the likelihood function (e.g., the likelihood function takes a multimodal form). The second problem often occurs due to an insufficient number of observations, which leads to reduction of the possibility of asymptotic inference. The third barrier comes when research interests concern the nonlinear function of the parameters. Standard methods in the face of complex dependencies between parameters do not allow formal inference about the uncertainty of nonlinear specification, because they require analytical results that are not always feasible. Within ZIP regression models, classical methods allow asymptotic estimation of uncertainty of regression parameters, but provide very rough information about uncertainty of the parameters p and λ.

When the problems presented above occur in the analysis and, at the same time, the interest of a study is to make formal inference about uncertainty without asymptotic properties or regardless of likelihood function shape, then the Bayesian inference is highly motivated.

BAYESIAN INFERENCE IN ZIP MODELS

Bayesian methods in demography are relatively rarely used, thus the opportunities they give are still not entirely familiarized. Bayesian analysis is often the only approach that allows researchers to obtain detailed analysis of the phenomenon in case of a small amount of data or in a situation when it is necessary to make inferences about non-linear functions of model parameters. In addition, this approach provides simple tools for effective forecasting and enables us to obtain covariates and their function distributions, as well as allows us to incorporate our *a priori* knowledge (from previous studies or experts' beliefs). A more detailed outline of Bayesian inference can be found in Koop or Osiewalski monograph [Polish version] (Osiewalski 2001, Koop 2003) and in studies of the applications of Bayesian methods in the field of financial econometrics (Zellner 1971, Bernardo and Smith 1994, Pajor 2003, Pipień 2006, Marzec 2008, Pajor 2010).

At first, the idea of Bayesian inference in demography was applied by Hyppola, Tunkelo, and Tornqvist, who applied a subjective approach to Finland population forecasting (Hgpölä et al. 1949). Due to the lack of computer power, which made Bayesian methods very laborious, the idea didn't spread among demographers at that point. But in 1986 and 1988, again the usefulness of Bayesian methods in demography was pointed out by Land and Pflaumer (Land 1986: 888–901, Pflaumer 1988: 135–142). Their studies encouraged other researchers to use a Bayesian approach. The idea has gained new followers and the popularity of Bayesian analysis among demographers has begun to increase slowly (Raftery 1995, Daponte et al 1997: 1256–1267, Bijak and Wiśniowski 2010, Osiewalski and Zając 2010: 77–86, Bijak 2011, Osiewalski and Zając 2011: 21–39, Bryant and Graham 2011, Raftery et al. 2012: 13915–13921, Bryant and Graham 2013).

Let us denote by $p(\theta)$ the prior knowledge about all unknown parameters. Posterior distribution is then formed from the *a priori* distribution and likelihood function of the model. In our study, the *a posteriori* distribution has the form presented below:

$$p(\theta \mid y) \propto L(\theta; y) p(\theta) =$$
$$= \left[\prod_{y_i=0} p_i \prod_{y_i>0} \left(\frac{(1-p)}{(1-\exp(-\lambda_i))} \exp(-\lambda_i) \frac{\lambda_i^{y_i}}{y_i!} \right) \right] f_N^{\gamma} f_N^{\delta} \tag{3}$$

where f_N^{γ} and f_N^{δ} are density functions for correspondent *a priori* distributions of γ and δ parameters.

Due to the unknown form of the posterior distribution[6], which is multidimensional, non-linear and too complicated to perform direct integration to determine its main characteristics, the Metropolis and Hastings (MH) algorithm was used. The procedure enables us to draw from the *a posteriori* distribution even when its form is analytically complicated.

The idea of the Metropolis and Hastings algorithm is to use some known, non-negative function $q(\theta^*;\theta^{(i-1)})$, called the proposal density, to generate a candidate state. Generally, the MH procedure consists of four subsequent steps [compare Geweke 1996, Robert and Casella 2005]:

1. Set up the initial point $\theta^{(0)}$ (it could be chosen arbitrary) and i=1.
2. Generate θ^* from the proposal density $q(\theta^*;\theta^{(i-1)})$ and u from the unitary distribution $U(0;1)$.
3. Check if the condition $\alpha(\theta^*,\theta^{(i-1)}) \geq u$ is fulfilled. If yes, set up $\theta^{(i)} = \theta^*$, otherwise ($\theta^{(i)} = \theta^{(i-1)}$). The $\alpha(\theta^*,\theta^{(i-1)})$, called the acceptance probability, has the following form:

$$\alpha\left(\theta^*, \theta^{(i-1)}\right) = \min\left\{ \frac{\pi\left(\theta^* \mid y\right) \cdot q\left(\theta^{(i-1)}; \theta^*\right)}{\pi\left(\theta^{(i-1)} \mid y\right) \cdot q\left(\theta^*; \theta^{(i-1)}\right)}, 1 \right\}, \tag{4}$$

where $\pi(\theta \mid y)$ is the kernel of the posterior density $p(\theta \mid y)$, so $\pi(\theta \mid y) \propto p(\theta \mid y)$.

When $q(\theta^*,\theta^{(i-1)})$ is a symmetric function of θ^* and $\theta^{(i-1)}$, the formula (4) can be rewritten as:

$$\alpha\left(\theta^*, \theta^{(i-1)}\right) = \min\left\{ \frac{\pi\left(\theta^* \mid y\right)}{\pi\left(\theta^{(i-1)} \mid y\right)}, 1 \right\}. \tag{5}$$

4. Set up i=i+1 and repeat point 2 and 3 M times.

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[6] Although the analytical form of posterior distribution is given by (3), still we are not able to "recognize" any known distribution with given (defined) characteristics. The formula in (3) is also too complicated to calculate directly the characteristics of the posterior distribution.

From a certain cycle i, the sample ($\theta^{(i)},\theta^{(i-1)},\ldots$) can be treated as a draw from the posterior distribution [Geweke 1996].

In this study we set up the proposal density for each MH step as the multivariate t-Student distribution with 4 degrees of freedom, that is:

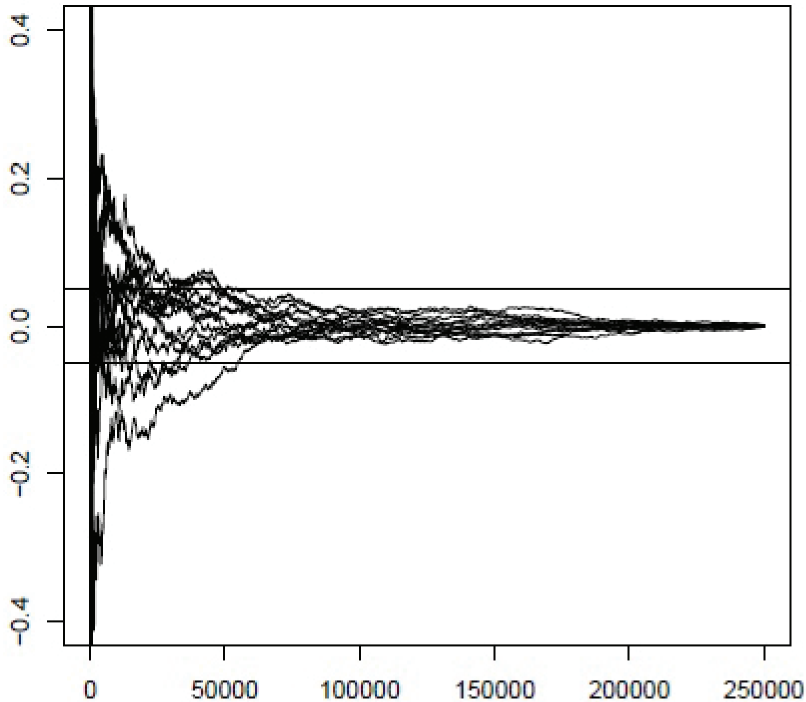
$$q\left(\theta^{*};\theta^{(i-1)}\right)=f_{St}^{k}\left(\theta^{(i-1)},\Sigma,4\right), \tag{6}$$

where k is the number of parameters in the zero or count states (in this case $k=7$ for both states) and Σ is an appropriately selected variance and covariance matrix. The choice of matrix depends on the researcher's preferences. This matrix is selected both to imitate in the best possible way the *a posteriori* distribution and to maintain the acceptance ratio at a reasonable level.

The problem presented in this study needed 100 000 initial cycles of the Metropolis and Hastings procedure (to "forget" the initial point, which usually is arbitrary chosen), then 150 000 burn-in cycles (to ensure convergence) and finally, 100 000 cycles considered as a (pseudo) random sample from the *a posteriori* distribution.

This number of burn-in cycles turned out to be sufficient to achieve convergence of the Metropolis and Hastings algorithm regarding to the CUSUM statistics proposed by Yu and Mykland (Yu and Mykland 1994). The CUSUM plot is presented in Figure 2. We can see that the cumulative sums for the final cycles (from 150 000 draw) do not exceed the interval (–0.05, 0.05). Therefore, it can be assumed that the number of cycles was sufficient to achieve the convergence and the final sample can be treated as draws from the stationary distribution. That is why in the next step the final cycles were used to determine marginal *a posteriori* distributions of $\gamma_1,\ldots,\gamma_s$ and $\delta_1,\ldots,\delta_r$ and calculate basic characteristics of the *a posteriori* distributions (expected value and standard deviation).

Figure 2. The convergence of CUSUM statistics for the MH algorithm



BAYESIAN MODELS COMPARISON

In order to build the most probable model that would reflect the empirical data and the initial knowledge in the best way, Bayesian methods provide us with a very intuitive formal model comparison framework (Jeffreys 1961, Greene 2003). As we introduce a probability measure on the model space, we can use the Bayes theorem to build a posterior on it. Posterior probabilities are calculated for each analyzed model according to the Bayes theorem:

$$p(M_i \mid y) = \frac{p(y \mid M_i) \cdot p(M_i)}{\sum_{j=1}^{m} p(y \mid M_j) \cdot p(M_j)}, \text{ for } i = 1, \dots, m, \tag{7}$$

where $p(y \mid M_i)$ is the marginal data density in the i-th model (or simply: probability of our data when model i is applied), and $p(M_i)$ is the prior probability of model *i* (conviction about model's accuracy). Then the posterior odds ratio of two competitive models should be calculated as follows:

$$PO = \frac{p(M_1 \mid y)}{p(M_0 \mid y)} = \frac{p(y \mid M_1)}{p(y \mid M_0)} \cdot \frac{p(M_1)}{p(M_0)} = Bayes\ \ factor\ \times\ prior\ \ odds\ \ ratio \tag{8}$$

The posterior odds ratio provides information on how many times more probable the first model is than model 0. For example, when PO=8, then model 1 is 8 times more probable than model 0. When the prior probability of model 1 is equal to the prior probability of model 0 (equal prior chances of each model), the posterior odds ratio simplifies to the Bayes factor (BF). Usually, the value of the BF is presented on a logarithmic scale. When $0<\log_{10}BF\leq 0.5$, we can say that the data very weakly testify against the model 0 (so the first model is more or less as good as model 0); when $0.5<\log_{10}BF\leq 1$, model 1 should be considered as better than model 0; when $1<\log_{10}BF\leq 2$, the data strongly pointed to model 1 as much better than model 0; and finally, when $2<\log_{10}BF$, the first model is definitely much more accurate than model 0 (Kass and Raftery 1995).

In the case that more than one model fits accurately to the data, then the next advantage of Bayesian methods could be taken – to include information given by each of the selected models, Bayesian pooling approach should be considered. The form of the posterior distribution is then as follows:

$$p(\theta \mid y) = \sum_{i=1}^{m} p_i(\theta \mid y) \cdot p(M_i \mid y), \tag{9}$$

where $p_i(\theta \mid y)$ is the posterior density, while model *i* is applied.

Usually (besides very basic models), to obtain the value of the Bayes factor that consists of two marginal data densities, numerical integration has to be performed. In this study, where only proper priors were used, to calculate the BF values Newton and Raftery's harmonic mean estimator (HME) for $p(y)$ was used (Newton and Raftery 1994):

$$p(y) \approx \left[\frac{1}{M} \sum_{i=1}^{M} \frac{1}{p(y \mid \theta^{(i)})} \right]^{-1}, \tag{10}$$

where $\theta^{(i)}$ is a pseudorandom sample from the posterior distribution. The HME, as shown by Newton and Raftery, is consistent. However, it should also be emphasized that this estimator has two major disadvantages: the first – it has no finite asymptotic variance, and the second – in small samples it overestimates the marginal data density (Lenk 2009, Pajor and Osiewalski 2013). That is why using HME could sometimes yield misleading results. Therefore, especially when the model comparison procedure does not point out clearly the differences between competing models, it is advised to use the adjusted HME with Lenk's correction (Lenk 2009). Nevertheless, in this study the standard HME will be used. The differences obtained with HME for the two competing models under consideration (ZIP and standard Poisson) was so big that including Lenk's correction should not change the conclusion. In turn, results for selection of covariates were confirmed by later results given by the posterior distribution (to obtain the posterior distribution, HME is not needed). Therefore, in this study using the standard HME could be justified. However, in the next studies we plan to introduce the adjusted HME estimator.

THE FORMATION OF REPRODUCTIVE BEHAVIORS AMONG AUSTRIAN WOMEN

VARIABLES DESCRIPTION

To analyze intergenerational transmission of reproductive behavior, the following demographic variables describing the mother's fertility pattern were chosen: number of respondent's siblings and mother's age at respondent's birth. It should be noted that instead of mother's age at respondent's birth, the more appropriate variable seems to be mother's age at first birth. Unfortunately, such information was not collected in GGS in Austria. Other variables were introduced in order to determine the basic daughter's characteristics, which also may have an impact on the analyzed number of children, but they are not main interest of this study (treated as control variables). These are: age (in years), educational level (highest educational level completed), type of settlement (urban or rural areas) and marital status (whether respondent was ever married). It should be mentioned that education, type of settlement and marital status are treated as fixed over time with the value as in the moment of interview. These characteristics in some cases could be different at the time when children were born. If the proper data are available, these changes could be introduced using the methods of Event History Analyses (EHA). However EHA concentrates mainly on the duration between selected events and not on the total number of those events. That is why, if the main interest of the study is to analyze the total number of births and not the duration between births, the ZIP model seems to be more convenient. To introduce the proper values of characteristics at the moment of birth of a child (when known), other covariates could be included to the ZIP model as well (e.g., the educational level at birth of a particular child). In some cases however, we should expect that the correlation between included covariates (e.g., education at birth of the first child and education at birth of the second child) will be probably very high. It could happen with covariates that generally do not change very often during lifetime (like education, being ever married, etc.). That is why the current values of characteristics such as education, marital status (treated as being ever married) or type of settlement, could be treated as an approximation of those values during the past life from a certain adult age, in case of lack of precise data.

Table 1 shows the structure of the sample population regarding the selected main and control variables.

The variables *number of siblings* and *mother's age at respondent's birth*, describe the fertility pattern developed in respondent's family home. From the data structure (Table 1), we can see that among the population of mothers, the typical number of children was two (so one sibling), while the largest number of children reached 14 (13 siblings). The most common age at respondent's birth was 23. The youngest mother at respondent's birth was 13 years old (3 women), and the oldest one – 46 years old (1 woman).

The first control variable *type of settlement* is divided into the following two levels: urban and rural areas. The possible differences within life styles in these two areas could influence the reproductive behaviors in a different manner. The majority of respondents from the studied population lived in urban areas (59.4%).

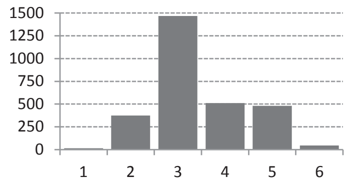
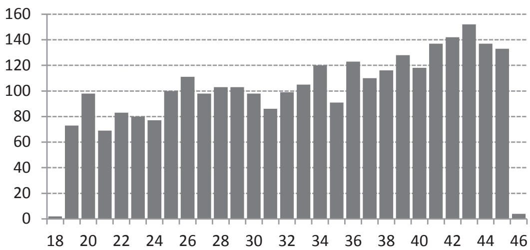
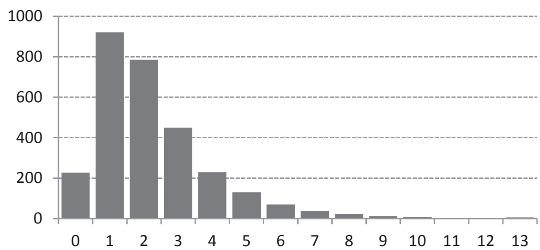
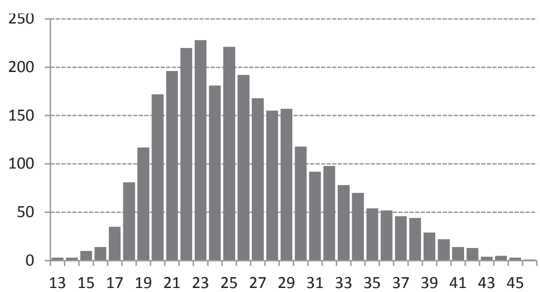
Within *education*, the six following levels are distinguished: primary, lower secondary, upper secondary, post-secondary, the first stage of tertiary, and the second stage of tertiary education. These levels are subject to the ISCED[7] classification adopted in the Austrian educational system. The introduction of this variable is justified by differences in reproductive behavior depending on education. The majority of daughters have upper secondary education (50.1%), while primary and second stage of tertiary education have only several respondents – respectively 15 women (0.5%) and 46 women (1.6%).

Marital status is divided into two levels, which split the population of women into those who have never been married and those who are married or were at least married once (so also widows and divorcees).

The population considered in this study (female respondents from the first wave of *Generations and Gender Study* in Austria in 2008 and 2009) was born between 1963 and 1990, so at the time of survey these respondents were between 18 and 46 years old. In this group, there were only two 18-year-old women and only four women who were 46. The number of respondents at the age from 19 to 45 was similar in each particular age, with a slight predominance of older ages.

[7] ISCED – International Standard Classification of Education designed by UNESCO

Table 1. Data structure

Variable	Level	Structure
Type of settlement	0 – rural 1 – urban	40.6% 59.4%
Education (ISCED codes)	1 – primary 2 – lower secondary 3 – upper secondary 4 – post secondary 5 – 1st stage of tertiary 6 – 2nd stage of tertiary	
Marital status	0 – never married 1 – at least married once	46.0% 54.0%
Age		
Number of siblings		
Mother's age at respondent's birth		

PRIOR MODEL ASSUMPTIONS AND POSTERIOR RESULTS

The need for a model that would allow analysis of a larger than expected number of zeros was supported by a formal Bayesian models comparison. The ZIP model was compared with the commonly used standard Poisson regression model. The prior probability was the same for each model (0.5 for ZIP and 0.5 for standard Poisson). The results clearly pointed that the ZIP model is much more adequate for the analyzed data – the decimal logarithm of the posterior odds ratio (in that case equal to the Bayes factor) is equal to 64. It means that the ZIP model is 10^{64} times more probable than the standard Poisson regression.

The ZIP model is based on γ and δ parameters (with dimensions s=r=7) for which the following *a priori* distributions were chosen:

$$\gamma \sim MVN\left(0_{[7x1]}, diag\left(1, 1, 1, 0.01, 1, 0.01, 1\right)\right) \tag{11}$$

$$\delta \sim MVN\left(0_{[7x1]}, diag\left(0.05, 0.05, 0.05, 0.0005, 0.05, 0.0005, 0.05\right)\right) \tag{12}$$

The hyperparameters of the *a priori* distributions were chosen to both enable all possible values and remain coherent with the common knowledge in the case of a hypothetical woman. It has to be emphasized that in fertility analyses of contemporary populations, the prior distribution should set higher probabilities for smaller numbers of children (from zero to 3) and at the same time very low probabilities (even equal zero) for numbers of children bigger than 15.

Let x be a covariate vector representing features of a chosen respondent, for example, x = (0, 4, 1, 30, 2, 23, 1) represents a woman who lives in rural area, has post-secondary education, is married, is 30 years old, has 2 siblings and her mother was 23 at the respondent's birth (1 on the last place stands for the intercept). Then, based on 10 000 draws from priors of γ and δ parameters, p and λ distributions for the chosen respondent were determined (Figure 3).

Subsequently, we specified the *a priori* distribution of number of children for the chosen woman (assuming prior distributions as previously). Results are presented in Figure 4. As we can see, the distribution of number of children assigns non-zero probability for all expected *a priori* values, therefore it seems to be a reasonable expression of our initial knowledge about the analyzed variable before looking into the data.

Figure 3. The p and λ distributions for a chosen woman

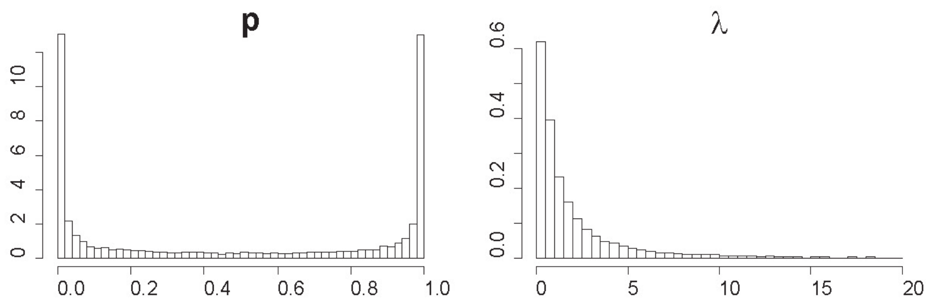
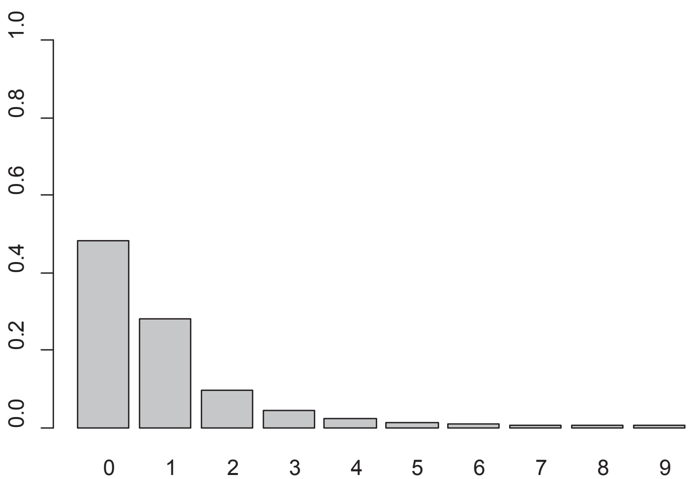


Figure 4. The prior distribution of number of children for a chosen woman



To build the final model, which has the most adequate set of covariates, and to check if there are some competitive models, Bayesian comparison was applied. Each time two models were compared: a full model (with all covariates) and a model without one chosen covariate. The prior chances for these models were always equal. When a covariate occurred to be negligible then the new "full model" is created (a model without this covariate) and the comparison starts again. The procedure is finished when there are no other negligible covariates. The decimal logarithms of the Bayes factor for the full model versus the model without one covariate are presented in the Table 2. The BF for the covariate *number of siblings* is equal 7.1. It means that the full model is *a posteriori* $10^{7.1}$ times more probable than the model without the *number of siblings*. The similar result is visible for *mother's* *age at respondent's birth* (BF equals 2.8). The smallest value of BF occurred for the control covariate *type of settlement*, but still the value 1.0 means that full model is better than model without this covariate. The analysis confirmed that the *a posteriori* most probable model is the model with all chosen covariates. There is no need for the Bayesian pooling approach[8].

Table 2. Logarithms of the Bayes factors

Variable	Logarithms of the Bayes factor (full model vs model without the covariate)
Type of settlement	1.008
Education	14.090
Marital status	118.835
Age	68.119
Number of siblings	7.069
Mother's age at respondent's birth	2.800

Marginal *a posteriori* distributions of all considered parameters are shown in Figures 5 and 6. The dotted line represents the corresponding prior in order to compare the two distributions and illustrate the strength of inference about the selected parameter based on the data. In turn, the dots mark 2.5% and 97.5% quantiles, which are helpful in determining the impact of the parameter on the modeled variable *number of children ever born*. If a zero value in the marginal *a posteriori* distribution of the parameter lies outside the interval set by the quantiles (so-called the highest posterior density interval – HPD), then it can be assumed that it has a significant impact on the analyzed phenomenon. However, if there is a substantial probability that the parameter can be equal zero (so zero belongs to the HPD interval) then its effect is treated as neutral or negligible.

It should be noted, that all obtained marginal posterior distributions compared to corresponding priors are more centralized around its expected value, otherwise the dispersion of these distributions is substantially lower. This fact leads to the conclusion that the inference about parameters is strong.

[8] To check formally that there is no need for knowledge pooling, the distributions from full model and model with the lowest BF (model without *type of settlement*) were combined. The full model was taken with the weight equal to $10^{BF}/(10^{BF}+1)\approx 0.91$; the second model had weight 0.09. The results given by pooled model were compared to the results obtained from the full model. There was no relevant differences.

Figure 5. The marginal posterior distributions – the zero model

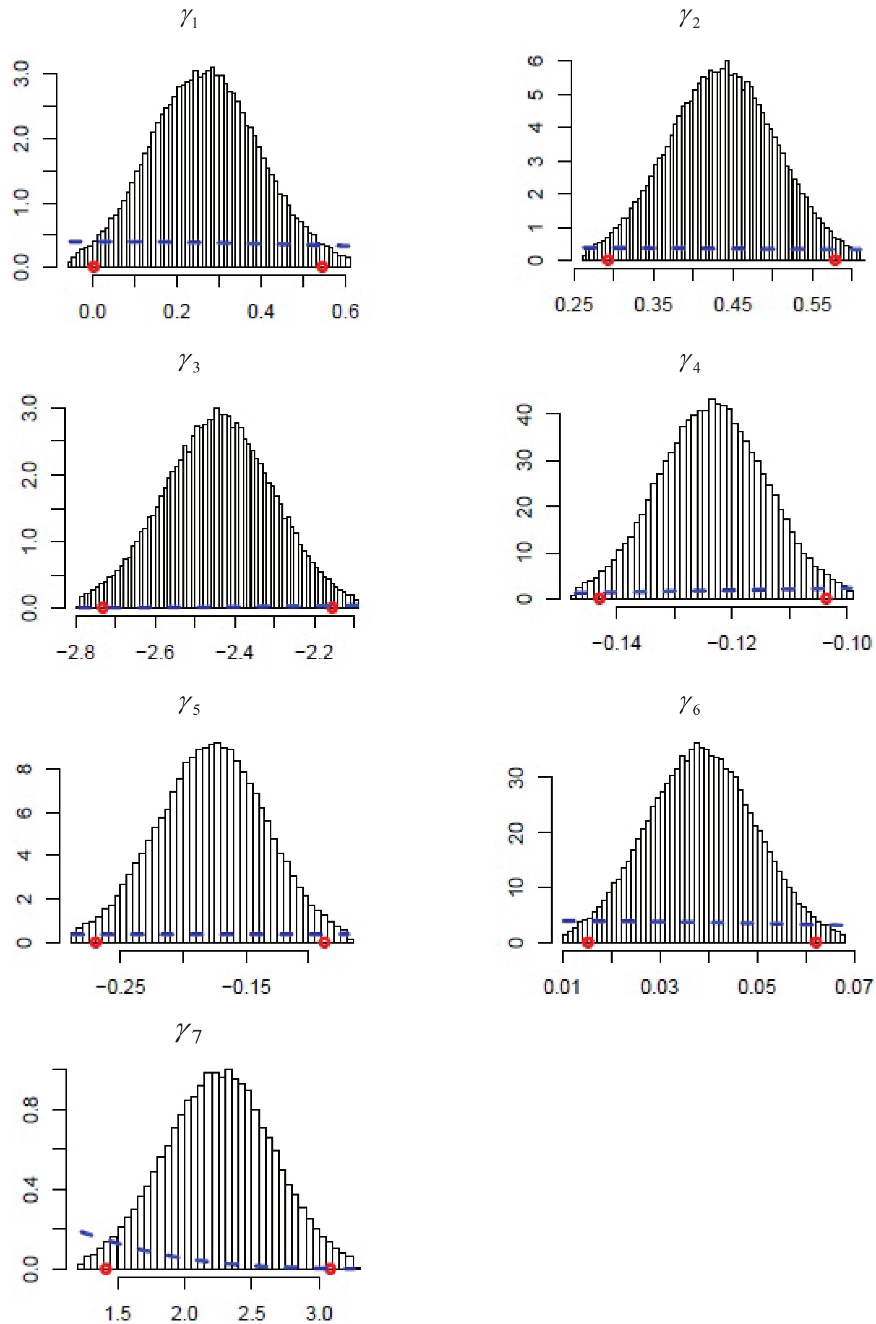
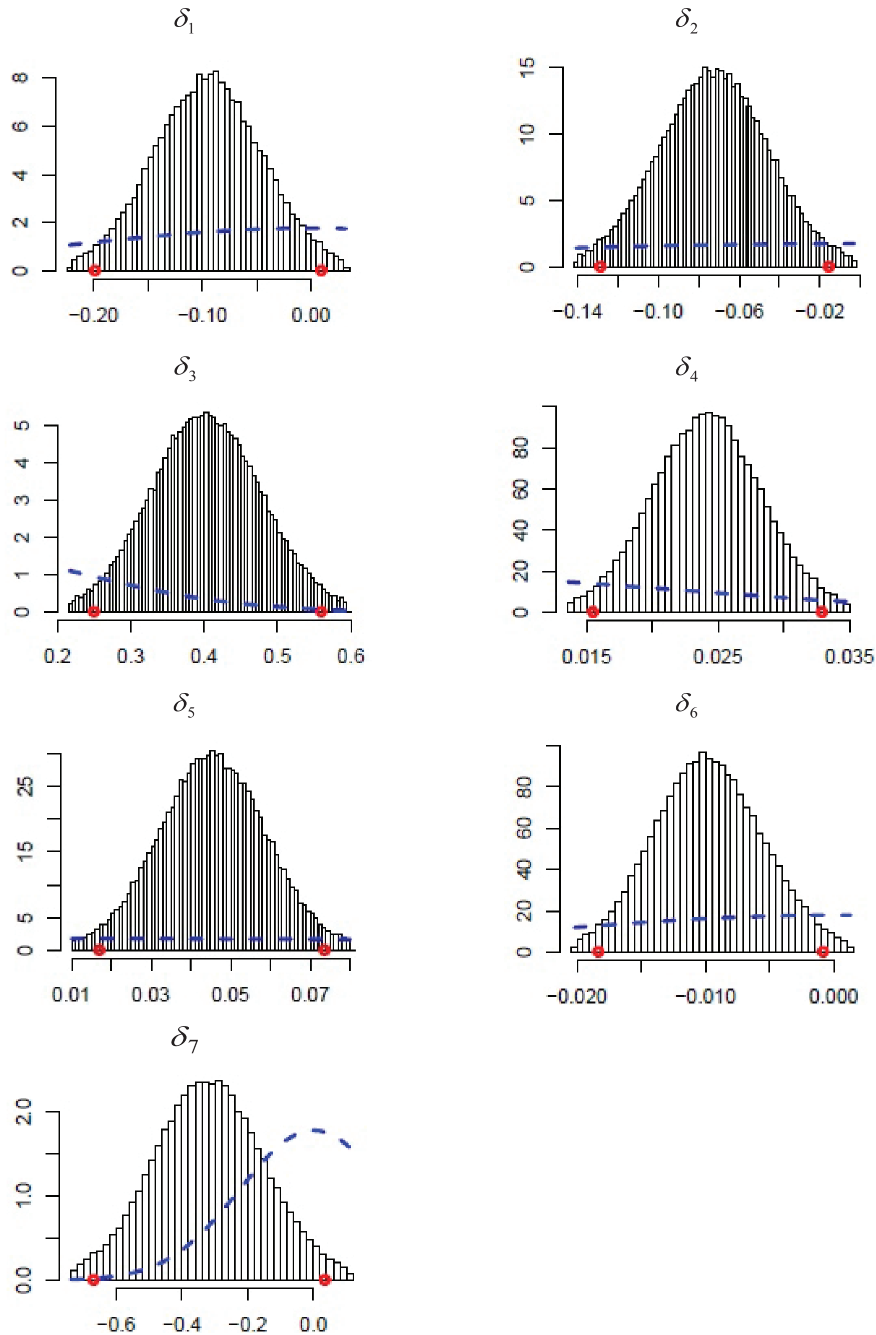


Figure 6. The marginal posterior distributions – the count model



The (pseudo) random sample obtained from the *a posteriori* distribution (final cycles) was used to determine the basic characteristics of the marginal distributions, such as the *a posteriori* expected value and *a posteriori* standard deviation. The results, presented in Table 3, are coherent with results obtained from the Bayesian model comparison: the *type of settlement* seems to be the least important in the analysis, while the strongest impact was *marital status*.

Table 3. The posterior expected values and standard deviations

	Zero model			Count model – Poisson		
Variable	Parameter	$E(\gamma \mid Y)$	$d(\gamma \mid Y)$	Parameter	$E(\delta \mid Y)$	$d(\delta \mid Y)$
Type of settlement	γ_1	0.268	0.138	δ_1	–0.095	0.053
Education	γ_2	0.436	0.073	δ_2	–0.073	0.029
Marital status	γ_3	–2.441	0.147	δ_3	0.403	0.080
Age	γ_4	–0.123	0.010	δ_4	0.024	0.004
Number of mother's children	γ_5	–0.177	0.046	δ_5	0.045	0.015
Mother's age at respondent's birth	γ_6	0.039	0.012	δ_6	–0.010	0.005
Intercept	γ_7	2.254	0.427	δ_7	–0.317	0.179

In case when zero belonged to HPD interval (between 2.5% and 97.5% quantiles) the value was marked with grey.

DAUGHTERS' FERTILITY ACCORDING TO CHOSEN CHARACTERISTICS

To specify the impact of selected covariates on the *a posteriori* expected number of children let us choose, as an example, one respondent by setting the value of j, j=1,…,n (in the studied data set n = 2896). For this purpose, let the respondent be a typical woman in the analyzed sample with the following characteristics: a woman living in the urban area with upper secondary education, who is or was married, has one sibling and her mother was 23 at the respondent's birth. Let x be a covariate vector representing features of this respondent, so x = (1, 3, 1, *age*, 1, 23,1). The age of the woman will be set as 25 and 45 to allow the comparison of a young woman who could just set the family and a woman who is almost at the end of her reproductive age.

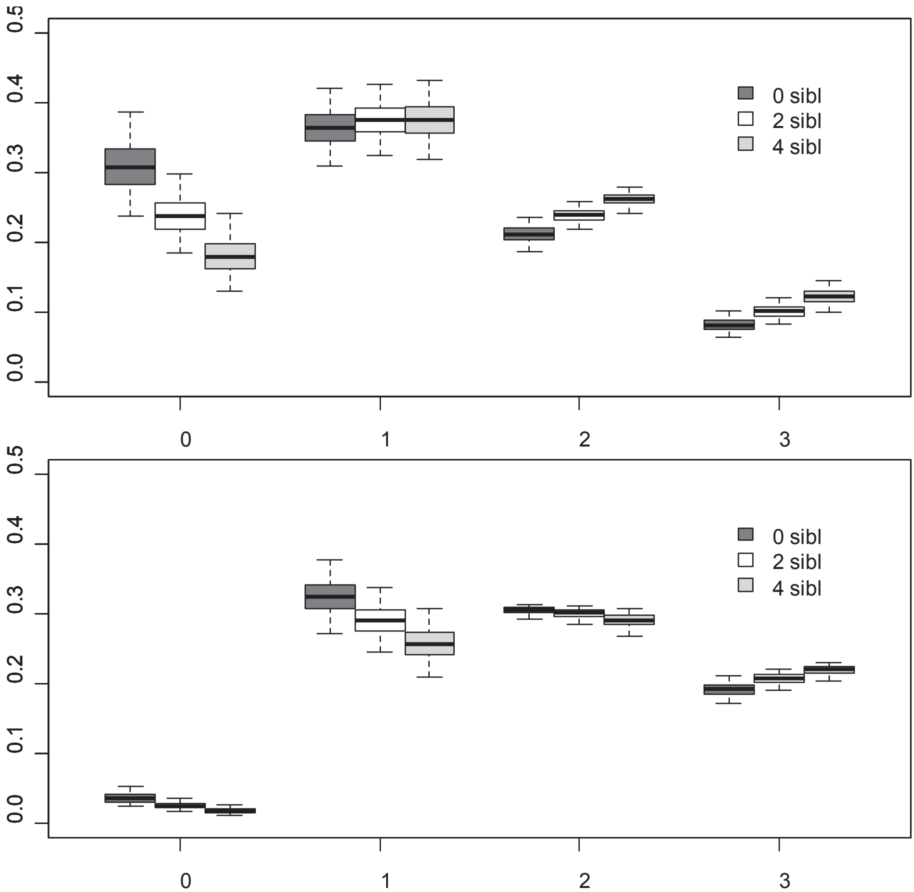
Let's have a look at covariates representing fertility patterns developed in the origin- family home. The first covariate, *number of siblings*, showed an important effect on a woman's procreative behavior. The *a posteriori* expected value of the parameter γ_5 is negative (–0.177) and positive for δ_5 (0.045). Therefore, we could say that the more siblings a woman has, the bigger family she will set. The posterior probabilities of a particular number of children due to the different numbers of siblings are shown in Figure 7. To make the figure more clear, only three different numbers of siblings (0, 2 and 4 siblings) and only numbers of children less than or equal to 3 are presented. The upper plot is for a 25-year-old woman, while the lower one is for a woman who is 45. Both plots are for a typical woman – all covariates are set except for the number of siblings: x = (1, 3, 1, 25/45, *number of siblings*, 23,1). In this, as in other figures of this type (so-called box-whiskers plot, Figures 7 and 8), a thick black line is the posterior median, rectangular boxes represent the interval set by the first and third quartile (25% and 75%), and whiskers cover the 2.5% and 97.5% quantile interval. The highest posterior probability for a 45-year-old woman who has siblings is for two children, while a woman who grew up as an only child would probably have one child. Additionally, it could be said that women with siblings decide to have a child at a younger age than women with no siblings: a 25-year-old woman is less probable to be still childlessness when she has siblings than when she is an only child.

The second fertility pattern covariate, *mother's age at respondent's birth* also significantly determines the number of a woman's children. Posterior probabilities for number of children due to mother's age at respondent's birth are presented in Figure 8. We can see that the older the mother was at the respondent's birth, the lower posterior probability that the daughter will have many children. A 25-year-old woman, whose mother was 45 at her birth, would probably be childless, while a woman of the same age whose mother was young (18 years old) at the woman's birth would already have one child. Finally, for a woman of age 45, the most probable number of children if her mother was 18 at her birth is two children, and if her mother was older at her birth, one child.

To summarize, the impact of both covariates *number of siblings* and *mother's age at respondent's birth* on the daughter's *number of children ever born*, reveal that her mother's fertility pattern affects a woman's procreative behavior.

Selected control variables generally appeared to be important for women's family models. The first control covariate – *type of settlement* has a slight impact on childlessness, but simultaneously no impact on parenthood was revealed. Education of a woman has a significant influence on both childlessness and parenthood: the more educated a woman is, the higher posterior probability of her childlessness and lower probability of having many children. The next covariate is *marital status*, which turned out to be very important in the analyzed population, its impact on *number of children ever born* is the strongest among all other covariates. Therefore, it could be said that marital status in Austria still strongly determines procreative behaviors.

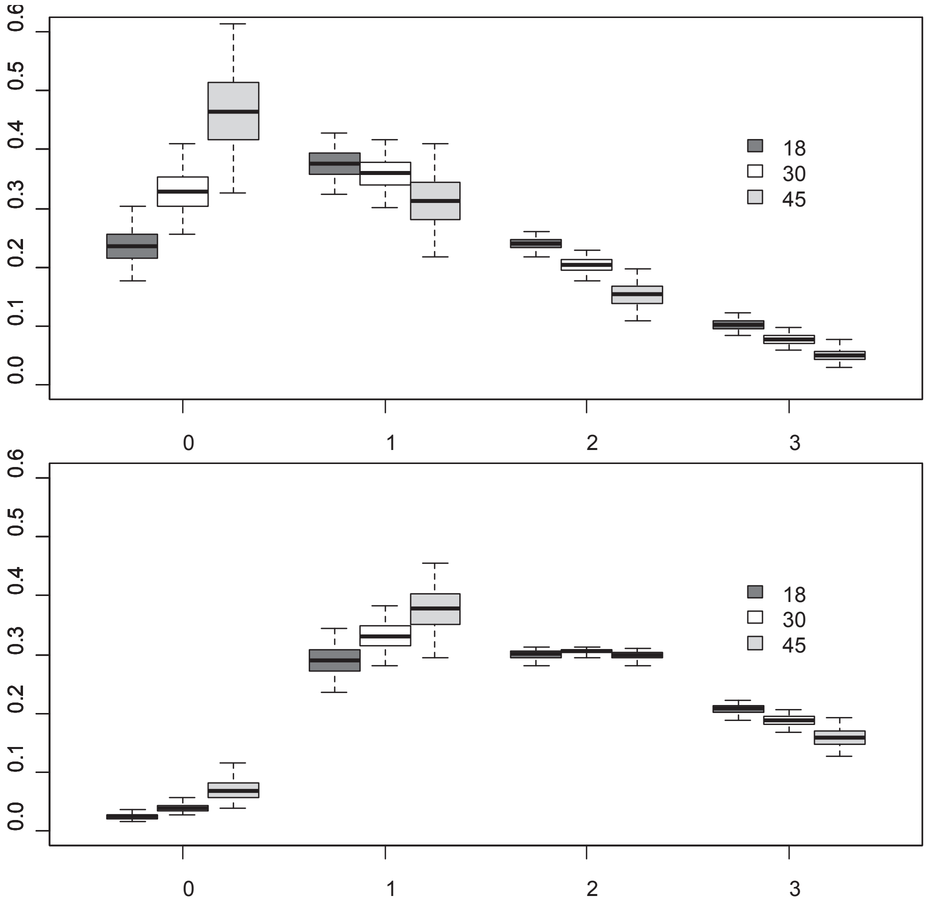
Figure 7. The posterior probabilities of particular number of children for a different number of siblings



Upper plot – 25 years old woman

Lower plot – 45 years old woman

Figure 8. The posterior probabilities of particular number of children for a different mother's age at respondent's birth



Upper plot – 25 years old woman

Lower plot – 45 years old woman

Parameters γ_7 and δ_7 represent intercepts in the zero and count model. The first one – γ_7 determines the "basic" value of probability of childlessness in case the other covariates in the analysis occur to be unimportant. Its *a posteriori* expected value is 2.254, thus the "basic" expected probability of zero equals 0.905. The second one – δ_7, which specifies the "basic" average number of children, turned out to be *insignificant*.

PROCREATIVE BEHAVIORS BY FEMALE PROFILES

In the previous section it was demonstrated that procreative behaviors are different due to social and demographic characteristics, such as age, educational level and marital status. What is more, this study revealed that the reproductive behaviors also depend on fertility patterns developed in the origin-family home. Therefore, determining women's procreative behaviors due to different female profiles seems to be valuable for such analyses.

To analyze the *a posteriori* distributions of the number of children for the chosen females' profiles, the Bayesian approach was crucial. Inference about these distributions would be impossible within the classical approach – we would not be able to obtain full knowledge about the distributions of nonlinear functions of parameters, such as the probability of childlessness or expected number of children.

In this study we defined five types of women. The first two profiles presented below are defined in order to demonstrate procreative behaviors of 35-year-old women with different family background:

A – *siblings profile* for typical women with different number of siblings: $x_A = (1, 3, 1, 35, \textit{siblings}, 23, 1)$;

B – *mother's age profile* for typical women with different age of mother at their birth: $x_B = (1, 3, 1, 35, 1, \textit{age}, 1)$.

The next three profiles are connected with social and demographic conditions and are presented for 25, 35 and 45-year-old women:

C – *typical woman* with the following characteristics (which occurred most often in studied dataset): living in urban area, with upper secondary education, married, has one brother or sister and her mother was 23 at the woman's birth, $x_C = (1, 3, 1, \textit{age}, 1, 23, 1)$;

D – *single educated woman*, so a respondent with characteristics as follows: living in urban area, with second stage of tertiary education, she has never been married, has no siblings and her mother was 28 at the woman's birth, $x_D = (1, 6, 0, \textit{age}, 0, 28, 1)$;

E – *married educated woman* – a respondent with characteristics like *single educated woman*, but married, $x_E = (1, 6, 1, \textit{age}, 0, 28, 1)$.

The differences in fertility patterns of types C and E are mainly due to the level of education, while types D and E are chosen to emphasize the strong impact of marital status on the *a posteriori* expected number of children.

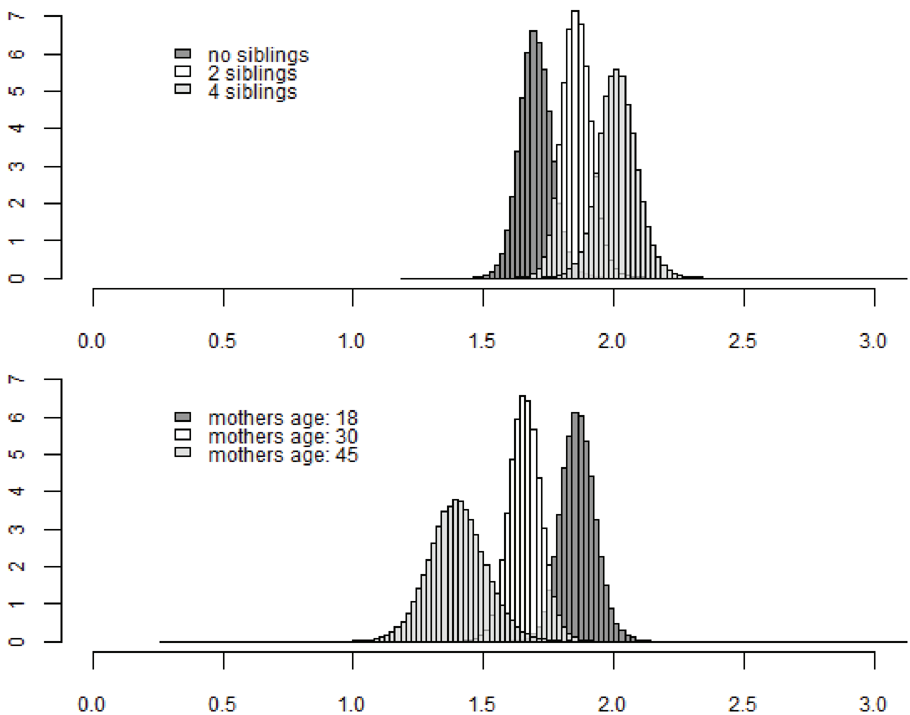
The posterior distributions of expected number of children for A and B female profiles are presented in Figure 9. The differences between women with other family background are visible and important. The posterior distribution of expected number of children due to *number of siblings* is presented in the upper plot in Figure 9. The following conclusion could be made: when a woman grew up in a big family, with many siblings, it is much more probable that she will follow the same pattern in her own family and decide to have more children, than a woman who grew up as an only child. A 35-year-old typical woman with 4 siblings would probably have 2 children (the *a posteriori* expected value is 2.014), while among women with no siblings one in three would have only one child (the *a posteriori* expected value is 1.699).

Additionally, *mother's age at respondent's birth* seems to have an important influence on a woman's procreative behaviors. The posterior distribution of expected number of children is presented in the lower plot in Figure 9. We can see that when a mother gave birth to a daughter at an older age, then the daughter will probably have fewer children. The *a posteriori* expected number of children for a 35-year-old typical woman, whose mother was 18 at her birth, is 1.783, and 1.316 for those whose mother was 45. This connection between daughters and mothers could be explained as follows: daughters compare their procreative behaviors with fertility patterns from their origin-family home, so when a mother gave birth to a daughter at her young age, then the daughter would earlier set her own family, so it is more probable that she will have more children.

Then, according to the next three profiles, the posterior distributions of the expected number of children are shown in Figure 10. Let's start from type C. The *a posteriori* expected number of children for a 25-year-old typical woman is equal to 1.261. In other words, among 25-year-old *typical women*, approximately one in four women have two children, while the other three have one child. For older women the proportion is changing. For a 35-year-old typical woman, the *a posteriori* expected number of children is equal to 1.777, and for a 45-year-old woman it reaches 2.228. It can be explained as follows: among 35-year-old women of type C, 75% already have two children, and 25% have one child. Finally, among the 45-year-old female population, three in four typical women have two children, and the other one in four have three children.

The next type is *single educated women* with the following characteristics: never married, fully educated woman who lives in an urban area. The posterior distribution of the expected number of children is presented in the middle plot in Figure 10. It can be said that almost for sure a young woman of type D would not have any children, since the *a posteriori* expected number of children is 0.057 and the distribution is strongly concentrated around its mean. For older women the deviations are bigger, but distributions are still strongly separated from 1. The posterior expected number of children for a 35-year-old *single educated woman* equals 0.189 and for a 45-year-old, 0.535. Thus, it can be said that 50% of 45-year-old women with D characteristics would have one child, while the other half would stay childless. Having more than one child is hardly probable here.

Figure 9. The posterior distributions of expected number of children for type A and B

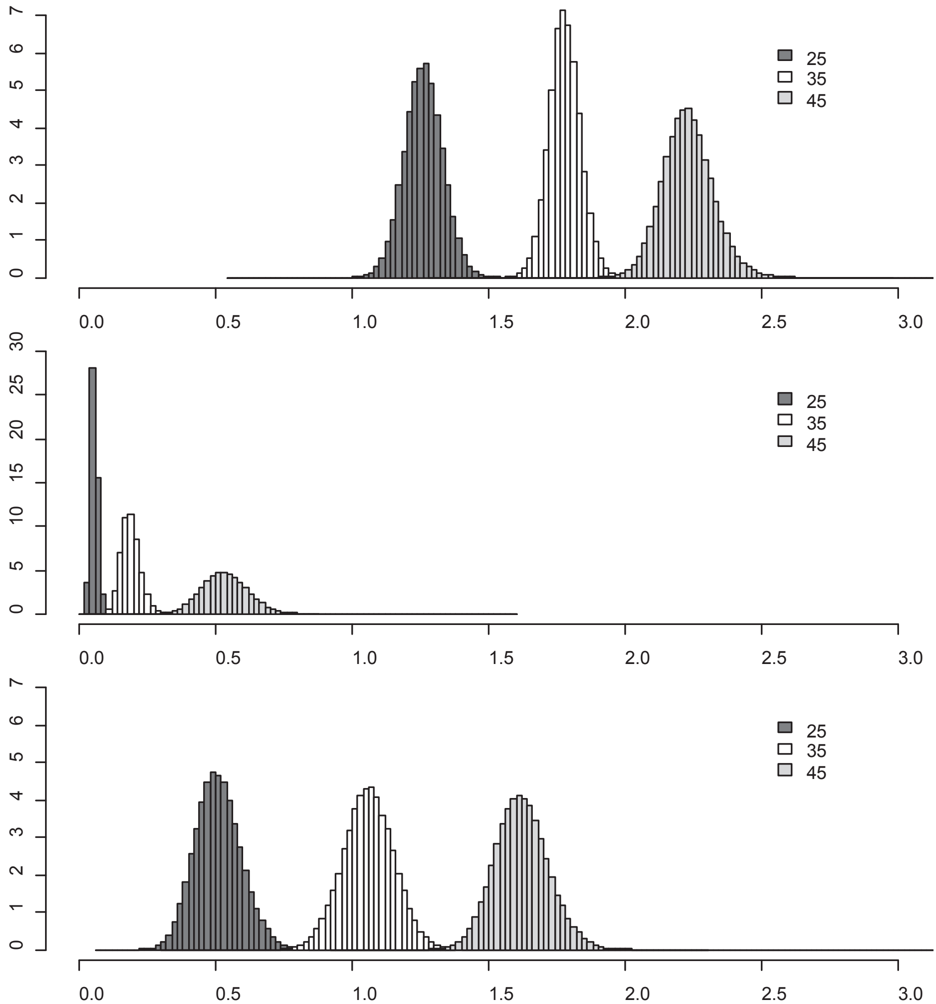


Upper plot – type A

Lower plot – type B

Finally, the posterior distribution of the expected number of children for *married educated woman* is presented in the lowest plot in Figure 10. These distributions compared with previous distributions for the D type (middle plot) show the strength of marital status. We can see that half of 25-year-old woman with E characteristics would already have one child (the *a posteriori* expected value is 0.509). A 35-year-old *married educated woman* would probably have one child, while among 45-year-old women, two in three would already have two children.

Figure 10. The posterior distributions of expected number of children for type C, D and E



Upper plot – type C

Middle plot – type D

Lower plot – type E

CONCLUSIONS

Based on the results of the research, general conclusions about the methodological and cognitive side of the study might be formulated. With regard to the method of analysis, it should be noted that the Zero-Inflated Poisson model and Bayesian inference form a useful framework for analyzing transmission of fertility patterns developed in the origin-family home. Within this study, the model allowed us to investigate the effect of fertility patterns developed in the origin-family home, as well as other selected demographic and social variables, separately on childlessness and parenthood in the population of daughters. Generally in fertility analyses, the Zero-Inflated Poisson model seems to be particularly useful when the special research interests are given to the total number of children (actual or intended), because the ZIP model gives a different qualitative dimension (value) to childlessness and parenthood. Within the ZIP model it is possible to analyze one set of covariates that could determine childlessness and another for parenthood (e.g., we can expect that enlarging family is influenced by the costs of rearing the previous children, while for childlessness these costs do not even exist). Simultaneously, we still keep these two states, which could be determined by different characteristics, under the one fertility model – so possible dependencies between those states (one state is connected to the other by the probability of childlessness/parenthood) are allowed.

In turn, the Bayesian approach enabled us to estimate the posterior distributions of having a certain number of children depending on the fertility pattern developed in origin-family home and other socio-demographic characteristics of a woman (distributions of expected number of children by female profiles). The uncertainty of the results are rather low, and the variances of the posterior distributions are small, so we are able to point out the differences clearly and the results are very precise.

Both components of the model (zero and count) were based on the same set of variables that characterize the mother's fertility pattern (number of children and age at respondent's birth) and social and demographic characteristics of daughters were included as control variables (age, type of settlement, education and marital status).

This methodological approach applied to GGS data for Austria have revealed that family procreative behaviors among contemporary populations are affected by the fertility pattern of the previous generation. Regarding the number of siblings, the results occurred to be coherent with the previous studies: there is observed the positive correlation between a mother's and a daughter's family size. Additionally, it was found that mother's age at daughter's birth is important for a daughter's number of children. The mechanism of the *mother-daughter fertility pattern transmission* could be explained as follow. Women compare her own family model with the pattern developed in the origin-family home. The origin-family pattern is likely to be treated as proper or safe (because it is known) and that is why it is likely to be repeated. Therefore, those females who grew up with siblings tend to have more children than women who grew up as an only child. The similar could be said about the age at childbearing. Women compare their *family calendar* with that of their mothers and when their mothers set families at young ages, they tend to adopt that pattern and, when possible (e.g., have partner, proper life conditions), set up own families sooner and consequently are more likely to have more children.

The following conclusions could be formulated:

1) Women adopt the fertility pattern developed in their family home. Among the Austrian female population the rule is as follows: having more siblings and a younger mother at one's own birth contribute to setting a bigger family oneself.
2) The origin-family pattern has strong influence on both the probability of being childless and the average number of children. Growing up as an only child increases the chance of childlessness by 16.2% as compared to having only one sibling, while within parenthood it decreases the average number of children by 4.6%. Having a k-years older mother (as compared to (k+1)-years older mother) decreases the chance of being childless by 3.8% and increases the average number of children by 1.0%.
3) Treating zero as a different state allows us to find that childlessness (in contrast to parenthood) is also influenced by the type of settlement – the chance of being childless is higher by 30.7 % in urban than in rural areas.

Special attention should be given to the constructed female profiles, which were called: *siblings profile*, *mother's age profile, typical woman*, *single educated woman* and *married educated woman*. The first two profiles (*siblings* and *mother's age*) were created to show differences in procreative behaviors among women with other family backgrounds. Thus, women who grew up as an only child and whose mothers were 45 at their birth, will probably have only one child, while women with 4 siblings and an 18-year-old mother at their birth, two children. The last three profiles present differences in procreative behaviors due to socio-demographic characteristics. In this domain the conclusions are as follows: *typical women* in Austria up to age 45 would already have two, sometimes three children. A woman representing the second profile (*single educated woman*) would stay childless or decide to have only one child. The third group of women (*married educated woman*) would probably have one child or two children.

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ABSTRACT

The connection between fertility of parents and their children has been investigated many times over the past century. It seems to be insignificant among pre-transitional populations, but becomes more important over time, especially in developed countries. Following Pearson's example, it was widely adopted to use simple correlation analyses in such studies. In this study we will present how to use more advanced statistical models and methods to determine the occurrence and strength of examined relationships. Thus, we aim to investigate the intergenerational transmission of fertility in contemporary populations (in the case of the mother-daughter relation in Austria) using the zero-inflated Poisson regression model. Using this model in fertility analysis allows us to treat childlessness as a qualitatively different state with possibly different determinants than parenthood (regardless of the number of children). Bayesian inference in this study enables us to obtain covariates' distributions as well as distributions of covariates' nonlinear functions (including their uncertainty) and allows us to incorporate our prior knowledge.

Keywords: fertility patterns, intergenerational transmission of fertility, Zero-Inflated Poisson, fertility modeling, Bayesian inference, fertility in Austria